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Closed inflationary universe with tachyonic field

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Abstract. In this article we study closed inflationary universe models by using a tachyonic field theory. We determine and characterize the existence of a universe which has $\Omega > 1$ and which describes a period of inflation. We find that the considered models are less restrictive compared to the standard ones with a scalar field. We use recent astronomical observations to constrain the parameters appearing in the model. The results obtained are compared to those found in the standard scalar field inflationary universes.

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1 Introduction

The existence of Doppler peaks and their respective localization tend to confirm the inflationary paradigm, associated with a flat universe with $\Omega \simeq 1$, as corroborated by the existence of an almost scale invariant power spectrum, with $n_s \sim 1$ [1–3].

The temperature anisotropy power spectrum, recently measured with the Wikinson microwave anisotropy probe (WMAP three-year data) at high multipoles, is in agreement with an inflationary Λ-dominated CDM cosmological model. However, the low order multipoles have lower amplitudes than expected from this cosmological model [4], and the mismatch of these amplitudes may indicate the need of new physics. Speculations for explaining this discrepancy have been invoked in the sense that the low quadrupole observed in the CMB is related to the curvature scale [5]. Also, the CMB data [6] alone place a constraint on the curvature, which is $\Omega_k = -0.037^{+0.033}_{-0.039}$. Additions of the LSS data yield a median value of $\Omega_k = -0.027 \pm 0.016$. Restricting H_0 by the application of a Gaussian HST prior, the curvature density determined from Boom2K flight data set and all previous CMB results is $\Omega_k = -0.015 \pm 0.016$. The constraint $\Omega_k = -0.010 \pm 0.009$ is obtained by combining the CMB data with the red luminous galaxy clustering data, which have their own signatures of baryon acoustic oscillations [7]. The WMAP three-year data can (jointly) constrain $\Omega_k = -0.024^{+0.016}_{-0.013}$ even when allowing for dark energy with an arbitrary (constant) equation of state w [3]. (The corresponding joint limit from WMAP three-year data on the equation of state is also impressive, $w = -1.062^{+0.128}_{-0.079}$.)

Due to these results, it may be interesting to consider other inflationary universe models in which the spatial curvature is taken into account [8]. In fact, it is interesting to check if the flatness in the curvature, as well as in the spectrum, is indeed reliable and robust predictions of inflation [9].

In the context of an open scenario, it is assumed that the universe has a lower-than-critical matter density and, therefore, a negative spatial curvature. Several authors [10–16], following previous speculative ideas [17, 18], have proposed alternative models in which open universes may be realized, and their consequences, such as density perturbations, have been explored [19]. The only available semi-realistic model of open inflation with $1 - \Omega \ll 1$ is rather unpleasant since it requires a fine-tuned potential of very peculiar shape [15, 16, 20]. The possibility to create an open universe from the perspective of the brane-world scenarios also has been considered [21, 22].

The possibility to have inflationary universe models with $\Omega > 1$ has been analyzed in [9, 23–27]. In this paper we would like to describe this kind of models.

One normally considers the inflation phase to be driven by the potential or vacuum energy of a scalar field, whose dynamics is determined by the Klein–Gordon action. However, more recently, and motivated by string theory, other non-standard scalar field actions have been used in cosmology. In this context the deep interplay between small-scale non-perturbative string theory and large-scale brane-world scenarios has aroused interest in a tachyon field as an inflationary mechanism, especially in the Dirac–Born–Infeld action formulation as a description of the D-brane action [28–33]. In this scheme, rolling tachyon matter is associated with unstable D-branes. The decay of these D-branes produces a pressureless gas with finite energy density that resembles classical dust. Cosmological implications of this rolling tachyon were first studied by Gibbons [34] and in this context it is quite natural to consider scenarios in which inflation is driven by the rolling tachyon.

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In recent years the possibility of an inflationary phase described by the potential of a tachyon field has been considered in a quite large diversity of topics [35–49]. In the context of an open inflationary scenario, a universe dominated by tachyon matter is studied in [50].

In this paper we adopt the point of view considered by Linde [9] but in which a tachyon field theory is considered. More precisely, we suppose that a closed universe appears from nothing at the point in which $\dot{a}=0$ and $\dot{\phi}=0$ and the potential energy density is $V(\phi)$. We solve the Friedmann and tachyon field equations, considering the acceleration of the universe to be sufficient for producing an inflationary period. It should be clear from the beginning that the tachyon potential considered by us satisfies $\mathrm{d}V/\mathrm{d}\phi<0$ for $\phi>\phi_0$ and $V(\phi\to\infty)\to 0$. On the other hand, we assume that the potential becomes extremely large in the vicinity of $\phi<\phi_0$, since the closed universe appeared at this point.

The paper is organized as follows. In Sect. 2 we review briefly the cosmological equations in the tachyon models. Section 3 presents a toy model in some detail. We get the value of the tachyon field when inflation begins. We also obtain the probability of the creation of a closed universe from nothing. In Sect. 4 we consider a model with a tachyonic exponential potential. In Sect. 5 the cosmological perturbations are investigated. Finally, in Sect. 6, we summarize our results.

2 Cosmological equations in the tachyon models

The action for our model is given by [51, 52]

$$S=S_{\mathrm{grav}}+S_{\mathrm{tach}} =\int\sqrt{-g}\,\mathrm{d}^4x\left[\frac{R}{2\kappa}-V(\phi)\sqrt{1-\partial^\mu\phi\partial_\mu\phi}\right], \tag{1}$$

where $\kappa=8\pi G=8\pi/M_{\mathrm{p}}^2$ (here M_{p} represents the Planck mass) and where $V(\phi)$ is the scalar tachyon potential.

The energy density ρ and pressure p for the tachyonic field are given by

$$\rho=\frac{V(\phi)}{\sqrt{1-\dot{\phi}^2}} \tag{2}$$

and

$$p=-V(\phi)\sqrt{1-\dot{\phi}^2}\,, \tag{3}$$

respectively.

The Friedmann–Robertson–Walker metric is described by

$$\mathrm{d}s^2=\mathrm{d}t^2-a(t)^2\,\mathrm{d}\Omega_k^2\,, \tag{4}$$

where $a(t)$ is the scale factor, t represents the cosmic time and $\mathrm{d}\Omega_k^2$ is the spatial line element corresponding to the hypersurfaces of homogeneity, which could be represented as a three-sphere, a three-plane or a three-hyperboloid, with values $k=1,0,-1$, respectively. From now on we will restrict ourselves to the case $k=1$ only. Using the metric (4) in the action (1), we obtain the following field equations:

$$\left(\frac{\dot{a}}{a}\right)^2=-\frac{1}{a^2}+\frac{\kappa}{3}\frac{V(\phi)}{\sqrt{1-\dot{\phi}^2}}\,, \tag{5}$$

$$\frac{\ddot{a}}{a}=-\frac{\kappa}{6}(\rho+3p)=\frac{\kappa}{3}\frac{V(\phi)}{\sqrt{1-\dot{\phi}^2}}\left(1-\frac{3}{2}\dot{\phi}^2\right), \tag{6}$$

and

$$\frac{\ddot{\phi}}{1-\dot{\phi}^2}=-3\frac{\dot{a}}{a}\dot{\phi}-\frac{1}{V(\phi)}\frac{\mathrm{d}V(\phi)}{\mathrm{d}\phi}\,, \tag{7}$$

where the dot over ϕ and a denotes the derivative with respect to time t. For convenience we will use the units in which $c=\hbar=1$.

3 Constant potential

In the spirit of [9], we study a closed inflationary universe in which the inflation is driven by a tachyon field. First, let us consider a simple tachyon model with the following step-like effective potential: $V(\phi)=V=\text{constant}$ for $\phi>\phi_0$, and where $V(\phi)$ is extremely steep for $\phi<\phi_0$. We consider the birth of the inflating closed universe to possibly be created "from nothing", in a state where the tachyon field takes the value $\phi_{\mathrm{in}}\le\phi_0$ at the point with $\dot{a}=0$ and $\dot{\phi}=0$, and the potential energy density in this point is $V(\phi_{\mathrm{in}})\ge V=\text{const.}$ If the effective potential for $\phi<\phi_0$ grows very sharply, then the tachyon field instantly falls down to the value ϕ_0, with potential energy $V(\phi_0)=V$, and its initial potential energy $V(\phi_{\mathrm{in}})$ becomes converted to the kinetic energy. Since this process occurs instantly, we can take $\dot{a}=0$, so that the tachyon field arrives at the plateau with a velocity given by

$$\dot{\phi}_0=+\sqrt{1-\left(\frac{V}{V(\phi_{\mathrm{in}})}\right)^2}\,. \tag{8}$$

Thus, in order to study the inflation in this scenario, we have to solve (6) and (7) in the interval $\phi\ge\phi_0$, with the initial conditions $\dot{\phi}=\dot{\phi}_0$, $a=a_0$ and $\dot{a}=0$. These equations have different solutions, depending on the value of $\dot{\phi}_0$. In particular, if we insert (8) into (6), we obtain

$$\frac{\ddot{a}}{a}=\frac{\kappa}{6}V(\phi_{\mathrm{in}})\left[3\left(\frac{V}{V(\phi_{\mathrm{in}})}\right)^2-1\right]. \tag{9}$$

Then we notice that there are three different scenarios, depending on the particular value of $V(\phi_{\mathrm{in}})$. First, in the particular case when

$$\frac{V}{V(\phi_{\mathrm{in}})}=\frac{1}{\sqrt{3}}\quad\text{or}\quad\dot{\phi}_0^2=\frac{2}{3}\,, \tag{10}$$

we see that the acceleration of the scale factor is $\ddot{a}=0$. Since initially $\dot{a}=0$, the universe remains static, and the tachyon field moves with the constant speed given by (8).

In the second case we have

$$0<\frac{V}{V(\phi_{\rm in})}<\frac{1}{\sqrt{3}} \quad \text{or} \quad \frac{2}{3}<\dot{\phi}_0^2<1\,. \tag{11}$$

In this case the universe starts moving with a negative acceleration ($\ddot{a}<0$) from the state $\dot{a}=0$. Then, in the tachyon field equation describing negative friction we have a term that makes the moving of ϕ even faster, so that $\ddot{a}$ becomes more negative. This universe rapidly collapses.

The third case corresponds to

$$\frac{1}{\sqrt{3}}<\frac{V}{V(\phi_{\rm in})}<1 \quad \text{or} \quad 0<\dot{\phi}_0^2<\frac{2}{3}\,. \tag{12}$$

In this case we have $\ddot{a}>0$, and the universe enters into an inflationary stage.

In what follows, we are going to make a simple analysis of the cosmological equations of motion for the cases where the condition (12) is satisfied. The tachyon field satisfies the equation

$$\frac{\ddot{\phi}}{1-\dot{\phi}^2}+3\frac{\dot{a}}{a}\dot{\phi}=0\,, \tag{13}$$

which implies

$$\dot{\phi}^2(t)=\frac{1}{1+Ca^6(t)}\,, \tag{14}$$

where C is a positive integration constant defined by

$$C=\frac{1-\dot{\phi}_0^2}{\dot{\phi}_0^2 a_0^6}\,, \tag{15}$$

and having $\dot{\phi}^2<1$.

Here $\dot{\phi}_0$ is the initial velocity of the field ϕ, immediately after it rolls down to the flat part of the potential. The effect of the tachyonic field in this model is reflected in the change of the slope of the tachyonic field ϕ, when compared to the standard case, where $\dot{\phi}=\dot{\phi}_0[a_0/a(t)]^3$.

The behavior of the tachyon field expressed by (14) implies that the evolution of the universe rapidly falls into an exponential regime (inflationary stage), where the scale factor becomes $a\sim \mathrm{e}^{Ht}$ with $H=\sqrt{\frac{\kappa V}{3}}$. When the universe enters the inflationary regime, the tachyon field moves by an amount $\Delta\phi_{\rm inf}$ and then stops. From (14) we get

$$\Delta\phi_{\rm inf}=\frac{1}{3H}\sinh^{-1}\left(\frac{1}{\sqrt{C}}\right) =\frac{1}{3H}\ln\left(\frac{1}{\sqrt{C}}+\sqrt{1+\frac{1}{C}}\right)\,. \tag{16}$$

Note that, when $\dot{\phi}_0\ll 1$ ($a_0=1$), we obtain $\Delta\phi_{\rm inf}\approx\frac{\dot{\phi}_0}{3H}$, which coincides with the result obtained in [9].

At an early time, before inflation takes place, we can conveniently write the equation for the scale factor as follows:

$$\ddot{a}(t)=\frac{2\kappa}{3}Va(t)\beta(t)\,. \tag{17}$$

Here, we have introduced a small time-dependent dimensionless parameter $\beta(t)$:

$$\beta(t)=\frac{1}{2}\frac{1}{\sqrt{1-\dot{\phi}^2}}\left(1-\frac{3}{2}\dot{\phi}^2\right)\,. \tag{18}$$

Certainly $\beta(t)\ll 1$ when $\dot{\phi}^2(t)\to 2/3$.

Now we proceed to make a simple analysis of the scale factor equation (6) and the tachyon (7) for $\beta(0)\equiv\beta_0\ll 1$.

At the beginning of the process, we have $a(t)\approx a_0$ and $\beta(t)\approx\beta_0$; then (17) takes the form

$$\ddot{a}(t)=\frac{2\kappa}{3}a_0V\beta_0\,, \tag{19}$$

and hence for small t the solution of this equation is given by

$$a(t)=a_0\left(1+\frac{\kappa\beta_0 V}{3}t^2\right)\,. \tag{20}$$

From (14) and (20) we find that at the time interval where $\beta(t)$ becomes twice as large as β_0, Δt_1 is given by

$$\Delta t_1=\left[\frac{\left(1-\dot{\phi}_0^2\right)^3}{2\kappa V\dot{\phi}_0^2}\right]^{1/2} \times\left[\left(2-3\dot{\phi}_0^2\right)+\frac{\dot{\phi}_0^2}{2}\left(11-6\dot{\phi}_0^2\right)+\frac{3}{2}\dot{\phi}_0^4\right]^{-1/2}\,. \tag{21}$$

Consequently the tachyonic field increases by the amount

$$\Delta\phi_1\sim\dot{\phi}_0\Delta t_1\sim\frac{1}{\sqrt{\kappa V}}\,, \tag{22}$$

where we have kept only the first term in the expansion of Δt_1. Note that this result depends on the value of V, i.e. the increase of the tachyonic field is less restrictive than the one used in the standard scalar field, in which $\Delta\phi_1=\text{const.}\sim -1/(2\sqrt{3\pi})$ [9]. After the time $\Delta t_2\approx\Delta t_1$, where now the tachyonic field increases by the amount $\Delta\phi_2\approx\Delta\phi_1$, the rate of growth of $a(t)$ also increases. This process finishes when $\beta(t)\to 1/2$. Since at each interval Δt_i the value of β doubles, the number of intervals $n_{\rm int}$ after which $\beta(t)\to 1/2$ is

$$n_{\rm int}=-1-\frac{\ln\beta_0}{\ln 2}\,. \tag{23}$$

Therefore, if we know the initial velocity of the tachyon, we can estimate the value of the tachyon field at which the inflation begins:

$$\phi_{\rm inf} \sim \phi_0 - \left(1 + \frac{\ln \beta_0}{\ln 2}\right) \frac{1}{\sqrt{\kappa V}} \,. \tag{24}$$

This expression indicates that our result for $\phi_{\rm inf}$ is sensitive to the choice of the particular value of the potential energy V, apart from the initial velocity of the tachyonic field ϕ immediately after it rolls down to the plateau of the potential energy.

Note that if the tachyon field starts its motion with a small velocity, the inflation begins immediately. However, if the tachyon moves with a large initial velocity the inflation is delayed; but once the inflation begins, it never stops. This can be explained by the constancy of the potential, and as we will see in the next section, this particular problem disappears when we consider a more realistic tachyon model.

We return to the description of a model of quantum creation for a closed inflationary universe model. The probability of the creation of a closed universe from nothing is given by [53]

$$P \sim \mathrm{e}^{-2|S|} = \exp\left(\frac{-\pi}{H^2}\right) \sim \exp\left(\frac{-3\pi}{\kappa V(\phi)}\right) . \tag{25}$$

We first estimate the conditional probability that the universe is created with an energy density equal to $\sqrt{3}V - \beta_0 V$. Assuming that this energy is smaller than $V(\phi_{\rm in}) = \sqrt{3}V$, for the probability we get

$$P \sim \mathrm{e}^{-2|S|} \sim \exp\left(-\frac{3M_{\rm p}^4}{8(\sqrt{3}-\beta_0)V} + \frac{3M_{\rm p}^4}{8\sqrt{3}V}\right) \sim \exp\left(-\frac{M_{\rm p}^4 \beta_0}{8V}\right) , \tag{26}$$

where we have used that $\dot\phi^2 \ll 1$. This implies that the process of quantum creation of an inflationary universe is not exponentially suppressed if $\beta_0 < 8V/M_{\rm p}^4$.

4 Exponential potential

Now we proceed with a more realistic case, a model in which the effective potential is given by

$$V(\phi) \simeq V_0 \mathrm{e}^{-\lambda\phi} \,, \tag{27}$$

where λ and V_0 are free parameters, and the parameter λ is related with the tachyon mass [35]; in the following we will take $\lambda > 0$ (in units $M_{\rm p}$). We will also assume that the effective potential sharply rises to indefinitely large values in a small vicinity of $\phi = \phi_0$; see Fig. 1.

We assume that the whole process is divided in three parts. The first part corresponds to the creation of the (closed) universe "from nothing" in a state in which the tachyon field takes the value $\phi_{\rm in} \le \phi_0$ at the point with $\dot a = 0$ and $\dot\phi = 0$, and where the potential energy is $V(\phi_{\rm in})$. If the effective potential for $\phi < \phi_0$ grows very sharply, then the tachyon field instantly falls down to the value ϕ_0, with potential energy $V(\phi_0)$, and the initial potential energy becomes converted to kinetic energy; see the previous section. Then we have

$$\dot\phi_0^2 = 1 - \left(\frac{V(\phi_0)}{V(\phi_{\rm in})}\right)^2 . \tag{28}$$

Following the discussion of the previous section we suppose that the following initial condition is satisfied:

$$V(\phi_0) < V(\phi_{\rm in}) < \sqrt{3}V(\phi_0) \,, \tag{29}$$

which guarantees that the model arrives at an inflationary regime. As it was mentioned previously, in all other cases the universe remains either static, or it collapses rapidly.

The second and third parts of the process are described by (6) and (7) in the interval $\phi \ge \phi_0$ with the initial conditions $\dot\phi = \dot\phi_0$, $a = a_0$ and $\dot a = 0$. In particular, the second part of the process corresponds to the motion of the tachyon field before the beginning of the inflation stage, and it is well described by the following approximation of the equations of motion:

$$\frac{\ddot\phi}{1-\dot\phi^2} = -3\frac{\dot a}{a}\dot\phi \,, \tag{30}$$

$$\ddot a = \frac{2\kappa}{3} a V(\phi)\beta(t) \,, \tag{31}$$

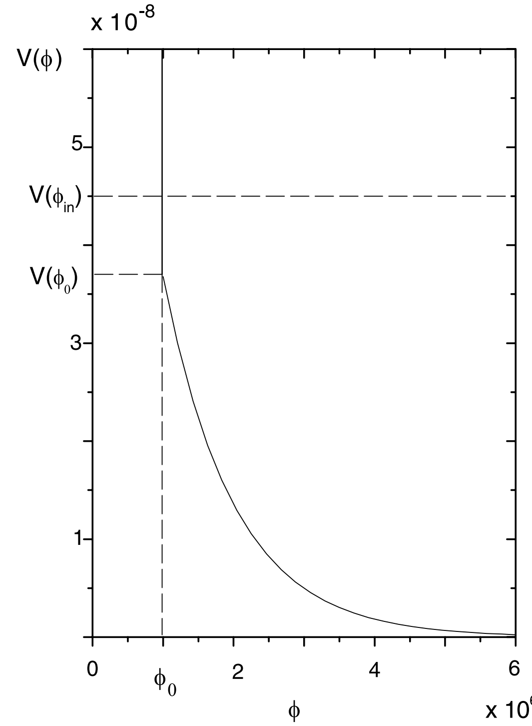

Fig. 1. The plot shows the tachyonic potential as a function of the tachyonic field ϕ. We have taken $V_0 = 10^{-7}\kappa^{-2}$ and $\lambda = 10^{-5}\kappa^{-1/2}$ in units with $\kappa = 1$

in which $\beta(t)$ satisfies $\beta(t) \ll 1$, as before.

The third part corresponds to the stage of inflation in which $\dot{\phi}$ is small enough and the scale factor $a(t)$ grows exponentially. This part is well described by the following approximation of the equation of motion [54]:

$$3\frac{\dot{a}}{a}\dot{\phi} = -\frac{1}{V}\frac{\mathrm{d}V}{\mathrm{d}\phi}\,, \quad (32)$$

$$\ddot{a} = \frac{\kappa}{3}aV(\phi)\,. \quad (33)$$

In summary, the whole process could be described as follows: the tachyon field starts its motion at $\phi = \phi_{\rm in}$, with $\phi_{\rm in} < \phi_0$; then the field immediately moves to the value ϕ_0 and acquires a non-null velocity $\dot{\phi}_0$. After that, the tachyon field starts to move subject to the equations of motion, and the potential in (7) can be neglected, so that the only contribution to the evolution of ϕ is the dissipative term. Therefore, during the period when the condition (29) is satisfied, ϕ satisfies (14), which means that $\dot{\phi}$ drops, while the size of the universe is increasing. This initial behavior for $\dot{\phi}$ is in very good agreement with the phase portrait (with numerical results) for tachyonic cosmology described in [55]. As a result of this process we arrive at an inflationary regime described by (32) and (33).

Now, we are going to describe the process in more detail. Let us consider the second stage, in which the tachyon field satisfies (30) and the scale factor satisfies (31). Following the scheme of Sect. 3 we solve the equation for $a(t)$ by considering $\beta(t) \ll 1$. Then at the beginning of the process, when $a \approx a_0$ and $\beta \approx \beta_0$, (31) takes the form

$$\ddot{a}(t) = \frac{2\kappa}{3}a_0 V(\phi_0)\beta_0\,, \quad (34)$$

and the tachyon field satisfies (14). The amount of increasing of the tachyon field during the time Δt that makes the value of β twice greater than β_0 is

$$\Delta\phi \approx \frac{1}{\sqrt{\kappa V(\phi_0)}}\,. \quad (35)$$

This process continues until $\dot{\phi}$ is small enough, so that the universe begins to expand in an exponential way, characterizing the inflationary era. We take the inflation to begin when $\beta(t)$ approaches $1/2$. Then, according to our previous result, the tachyon field gets the value

$$\phi_{\rm inf} \sim \phi_0 - \left(1 + \frac{\ln\beta_0}{\ln 2}\right)\frac{1}{\sqrt{\kappa V(\phi_0)}}\,. \quad (36)$$

In order to find an analytical solution to the equation of the tachyon field in the inflationary era, we are going to focus on the approximation of flat space for the Friedmann equations. Then we can use the result of [54].

The tachyon field satisfies the following equation:

$$\dot{\phi} = \frac{\lambda}{3\gamma}\mathrm{e}^{\lambda\phi/2}\,, \quad (37)$$

where $\gamma^2 = V_0\kappa/3$. Notice that $\dot{\phi}$ increases during the inflationary era; this is different from $\dot{\phi}$ in the previous period; see (14). The scale factor has the following behavior during this period:

$$\frac{a(t)}{a_i} = \mathrm{e}^{\gamma t[C-(\lambda^2/12\gamma)t]}\,, \quad (38)$$

where a_i is the value of the scale factor at the beginning of inflation and $C = \mathrm{e}^{-\frac{\lambda\phi_{\rm inf}}{2}}$. From (38) we notice that the scale factor passes through an inflection point, which marks the end of inflation. This happens when $\dot{\phi}_{\rm end} = \sqrt{2/3}$, which implies that the value of the potential at the end of inflation is $V_{\rm end} = \frac{\lambda^2}{2\kappa}$. The values of the tachyon potential at the beginning and at the end of inflation are related by the number of e-folds $\mathcal{N}$ [54]:

$$(2\mathcal{N}+1)V_{\rm end} = V(\phi_{\rm inf})\,. \quad (39)$$

Then, by using (36) and (39), we can relate the value of β_0 with the number of e-folds, $\mathcal{N}$:

$$\beta_0 = \frac{1}{2}\left[\frac{(2\mathcal{N}+1)}{V(\phi_0)}\frac{\lambda^2}{2\kappa}\right]^{\frac{\sqrt{\kappa V(\phi_0)}}{\lambda}\ln(2)}\,. \quad (40)$$

Note that, just as before, if the tachyon field starts its motion with a sufficiently small velocity (large β), inflation begins immediately and we have a large number of e-folds. On the other hand, if the tachyon field starts with a large initial velocity $\dot{\phi}_0 \sim \sqrt{2/3}$, corresponding to $\beta_0 \approx 0$, the beginning of inflation takes more time and we obtain a lower value of $\mathcal{N}$. Eventually, if β_0 is too small, we can arrive at the situation where

$$V(\phi_{\rm inf}) < V_{\rm end} = \frac{\lambda^2}{2\kappa}\,, \quad (41)$$

and the universe can never inflate.

As an example, we take a particular set of the parameters appearing in the tachyon potential (27). We also use the COBE normalized value for the amplitude of the scalar density perturbations in order to evaluate λ [54]; thus, we have $\lambda = 10^{-5}\kappa^{-1/2}$ and we take $V_0 = 10^{-7}\kappa^{-2}$.

Note that, if the field starts with a large velocity $\dot{\phi}_0$, the universe starts to inflate at a late time, and a lower value of the number of e-folds is obtained (see Fig. 2).

Now, let us analyze the quantum probability of the creation of the sort of universe considered. From the discussion of Sect. 3 we know that the probability for the universe with $\beta_0 \neq 0$ will be exponentially suppressed, unless the universe is created very close to the threshold value $V(\phi_{\rm in}) = \sqrt{3}V(\phi_0)$, with

$$\beta_0 < \frac{\kappa^2 V(\phi_0)}{8\pi^2}\,. \quad (42)$$

If we assume that $\phi_0 \sim 10^5\kappa^{1/2}$, then in order to satisfy (42) we have $\beta_0 < 2.2\times10^{-10}$. Following [9] we can argue that the probability for a start with the value $\beta_0 \ll 2.2\times10^{-10}$ is suppressed, due to the small phase space corresponding to these values of β_0. Thus, it is most probable to have $\beta_0 \sim 2.2\times10^{-10}$, and in that case, if we set

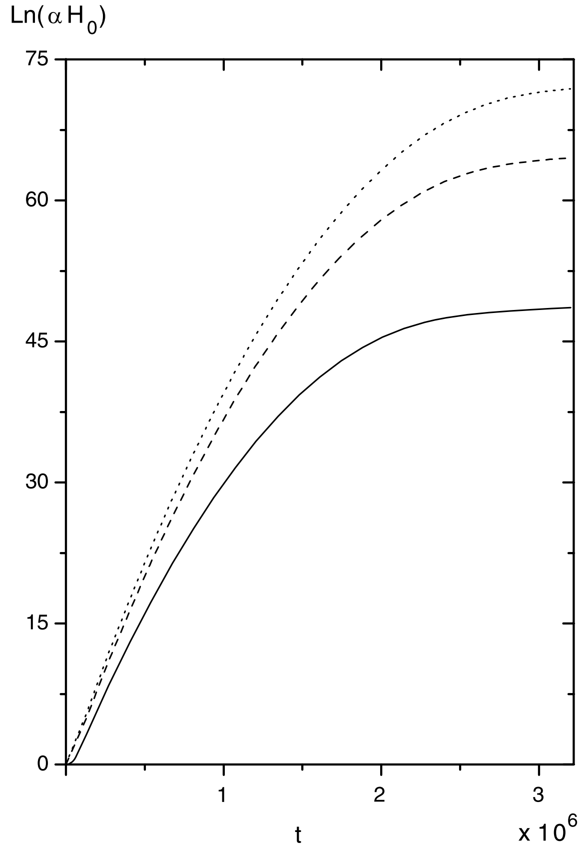

Fig. 2. This plot shows the number of e-folds as a function of cosmological time t, for different initial values of $\dot{\phi}_0^2$. We have taken $\kappa = 1$

$\phi_0 = 1.1 \times 10^5 \kappa^{1/2}$, which satisfies the condition (42), we obtain $\mathcal{N} = 60$, and this leads to $\Omega = 1.1$. On the other hand, if we take $\phi_0 = 0.5 \times 10^5 \kappa^{1/2}$, we get $\mathcal{N} = 171$ and the universe becomes flat.

5 Perturbations

Even though the study of scalar density perturbations in closed universes is quite complicated, it is interesting to give an estimation of the standard quantum scalar field fluctuations in this scenario. In particular, the spectra of the scalar perturbations for a flat space, generated during tachyon inflation, expressed in terms of the slow-roll parameters defined in [56], become [57]

$$\frac{\delta\rho}{\rho} = [1 - 0.11\epsilon_1 + 0.36\epsilon_2]\, \frac{\kappa H}{2\pi\sqrt{2\epsilon_1}}\,, \tag{43}$$

where the slow-roll parameters are given by

$$\epsilon_1 \simeq \frac{1}{2\kappa}\frac{(V_{,\phi})^2}{V^3}\,, \tag{44}$$

$$\epsilon_2 \simeq \kappa^{-1}\left[-2\frac{V_{,\phi\phi}}{V^2} + 3\frac{(V_{,\phi})^2}{V^3}\right]\,. \tag{45}$$

Certainly, in our case, (43) is an approximation and must be supplemented by several different contributions in the context of a closed inflationary universe [9]. However, one may expect that the flat-space expression gives a correct result for $N > 3$.

If one interprets the perturbations produced immediately after the creation of the closed universe (at $N \sim O(1)$) as perturbations on the horizon scale $l \sim 10^{28}$ cm then the maximum at $N \sim 10$ would correspond to the scale $l \sim 10^{24}$ cm, and the maximum at $N \sim 15$ would correspond to the scale $l \sim 10^{22}$ cm, which is similar to the scale of a galaxy.

One interesting parameter to consider is the so-called spectral index n_{s}, which is related to the power spectrum of the density perturbations, $P_{\mathcal{R}}^{1/2}(k)$. For modes with a wavelength much larger than the horizon ($k \ll aH$), the spectral index n_{s} is an exact power law, expressed by $P_{\mathcal{R}}^{1/2}(k) \propto k^{n_{\mathrm{s}}-1}$, where k is the comoving wave number. It is also interesting to give an estimate of the tensor spectral index n_{T}. In tachyonic inflationary models, the scalar spectral index and the tensor spectral index are given by

$$n_{\mathrm{s}} = 1 - 2\epsilon_1 - \epsilon_2\,, \tag{46}$$

and $n_{\mathrm{T}} = -2\epsilon_1$, in the slow-roll approximation [57].

One of the features of the three-year data set from WMAP is that it hints at a significant running in the scalar spectral index $\mathrm{d}n_{\mathrm{s}}/\mathrm{d}\ln k = \alpha_{\mathrm{s}}$ [3]. From (46) we obtain that the running of the scalar spectral index for our model becomes

$$\alpha_{\mathrm{s}} = \frac{\mathrm{d}n_{\mathrm{s}}}{\mathrm{d}\ln k} \simeq 2\frac{V\epsilon_1}{V_{,\phi}}[2\epsilon_{1,\phi} + \epsilon_{2,\phi}]\,, \tag{47}$$

where we have used that $\mathrm{d}\ln k = -\mathrm{d}N$. Using the exponential potential from (47), we find that

$$\alpha_{\mathrm{s}} = \frac{\mathrm{d}n_{\mathrm{s}}}{\mathrm{d}\ln k} \simeq -2\frac{\lambda^4}{\kappa^2 V_0^2}\mathrm{e}^{2\lambda\phi} = -2\frac{\lambda^4}{\kappa^2 V(\phi)^2}\,. \tag{48}$$

Note the difference that occurs with respect to a standard scalar field (with an exponential potential) where $\alpha_{\mathrm{s}} = 0$, since $n_{\mathrm{s}} = Cte. = 1 - M_{\mathrm{P}}^2\lambda^2$ [58].

In models with only scalar fluctuations, the marginalized value for the derivative of the spectral index is approximated by $\mathrm{d}n_{\mathrm{s}}/\mathrm{d}\ln k = \alpha_{\mathrm{s}} \sim -0.05$ for the WMAP three-year data only [3].

Note that, from (48), the scalar potential becomes $V(\phi_*) \sim 6.3\lambda^2/\kappa$, where ϕ_* represents the value of the tachyon field when the scale $k_0 = 0.002\,\mathrm{Mpc}^{-1}$ leaves the horizon. For $\lambda \sim 10^{-5}\kappa^{-1/2}$, we see that the scalar potential, when the scale is k_0 was leaving the horizon, becomes $V(\phi_*) \sim 10^{-9}\kappa^{-2}$. This value of the scalar potential is in agreement with [59], in which a chaotic potential with a standard scalar field is used.

Using the WMAP three-year data [3] and the SDSS large-scale structure surveys [60], an upper bound for $\alpha_{\mathrm{s}}(k_0)$ has been found, where $k_0 = 0.002\,\mathrm{Mpc}^{-1}$ corresponds to $L = \tau_0 k_0 \simeq 30$, with the distance to the decoupling surface being $\tau_0 = 14\,400\,\mathrm{Mpc}$. SDSS measures galaxy distributions at red-shifts $a \sim 0.1$ and probes k in the range $0.016\,\mathrm{h\,Mpc}^{-1} < k < 0.011\,\mathrm{h\,Mpc}^{-1}$. The recent WMAP three-year data results give the values for the scalar curvature spectrum $P_{\mathcal{R}}(k_0) \equiv 25\delta_H^2(k_0)/4 \simeq 2.3 \times 10^{-9}$ and the spectral index $n_s \simeq 0.95$. These values allow us to find the constraints on the parameters of our model. Furthermore, from the numerical solution we can obtain their values. In particular, for $k_0 = 0.002\,\mathrm{Mpc}^{-1}$, we have $n_s \approx 0.96$ and $n_T \approx -0.03$. Notice that those indices are very close to the Harrison–Zel'dovich spectrum [61, 62].

6 Conclusion and final remarks

In this work we have studied a closed inflationary universe model in a tachyonic field theory. In the context of Einstein's GR theory, this model was studied by Linde [9]. Firstly, we have assumed the tachyon scalar potential to be constant. Secondly, we have analyzed a closed universe model with a tachyon exponential potential. For these two parts, in which the potential is very sharp at small values of the field, $\phi < \phi_0$, we have found extra ingredients in the tachyonic theory, compared to its analog in standard scalar field theory. Specifically, we obtain a large stage of initial inflation for a closed universe, and this stage depends on the value of the potential energy V. In this way, we have found that our model, which takes into account a tachyonic theory, is less restrictive than the one used in standard scalar field theory.

Also, we have found that, after the tachyon field starts moving subject to the equations of motion, the potential term in (7) becomes irrelevant, and the only contribution to the evolution of ϕ is the dissipative term. Therefore, during this period, ϕ satisfies (14), which means that $\dot{\phi}$ drops, since the size of the universe is increasing in this period, if the condition (29) is satisfied. This initial behavior for $\dot{\phi}$ is in good agreement with the phase portrait (with numerical results) for the tachyonic cosmology described in [55].

We have also found that, for an exponential potential, the inclusion of the tachyonic field changes some characteristics of the running spectral index α_s, and it becomes $\alpha_s \neq 0$, in contrast to the standard case in which $\alpha_s = 0$. From the normalization of the WMAP three-year data, the potential becomes of the order of $V(\phi_*) \sim 10^{-10}\, M_P^4$ when it leaves the horizon at the scale of $k_0 = 0.002\,\mathrm{Mpc}^{-1}$. In particular, we expect that the Planck mission will significantly enhance our understanding of α_s by providing high quality measurements of the fundamental power spectrum over a large wavelength range compared to that of WMAP. Summarizing, we have been successful in describing a closed inflationary universe in a tachyon field theory.

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